应用数学讲座

Научный Семинар по Прикладной Математике

Research Seminar on Applied Mathematics

应用数学报告（58）

报告人 / Докладчик / Speaker: Associate Prof. Shishlenin M.（新西伯利亚大学）

题目 / Название / Title: Solving inverse problems for nonlinear equations of the reaction-diffusion-advection type with data on the position of a reaction front

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地点 / Место / Venue: Zoom ID: 462 476 1414
Password: 777777

摘要 / Аннотация / Abstract:

Problems for nonlinear singularly perturbed reaction-diffusion-advection equations arise in gas dynamics, combustion theory, chemical kinetics, biophysics, medicine, ecology, finance and other fields of science. A specific feature of problems of this type is the presence of processes of different scales. Therefore, the mathematical models of these problems are described by nonlinear parabolic equations with a small parameter at the highest derivative. In this regard, solutions to these problems may contain narrow moving fronts that divide the space into two parts: the disturbed part, through which the front has already passed, and the undisturbed part. The front is a region in which the function describing some characteristic of the medium (temperature, density, etc.) changes quite sharply from the values of the function describing one state of the medium (for example, undisturbed) to the value of the function describing another state. If there is a small parameter with the highest derivative, the width of such a front will be rather small in relation to the size of the entire region. As a consequence, the reaction front can be sometimes distinguished experimentally.

Some applied problems for equations of this type require solving inverse problems for recovering some coefficient in the equation. To formulate the inverse problem, additional information is required, which is usually measured in an experiment. Often, in the formulation of inverse problems for partial differential equations, additional information about the solution on a part of the boundary of the domain is used. However, one of the possible statements of inverse problems for equations of the type under consideration is a statement with additional information about the dynamics of the reaction front motion (see, for example, [1, 2, 3, 4]). Additional data of this type are

in demand in practice, since they are most easily to observe in an experiments (the front is an easily distinguishable contrast structure).

The simplest formulation of an inverse problem of this type relates to the case of recovering a function of an argument from experimental observations of a function of the same argument. For example, in [2], an approach was considered to restore the function of a temporary variable argument from the observation data of the function also of the variable argument in time. More complex formulations are formulations in which it is required to restore the function of one argument (for example, spatial) from the observation data of the function of another argument (for example, temporal) [1, 3, 4]. This class of inverse problems is considered recently -- the recovering of the function of the argument of a spatial variable (that determines the properties of the medium) from the data of observations of the function of the argument of the time variable (that determined by the dynamics of the reaction front).

To effectively solve such inverse problems the methods of asymptotic analysis can be used (sometimes!). In such cases, it is possible to reduce the original inverse problem for a nonlinear singularly perturbed partial differential equation to a much simpler problem with respect to the coefficient to be restored. The resulting simplified problem is called as a reduced statement (formulation) of the inverse problem. However, the reduced formulations of such inverse problems may have special features. It was shown that the reduced formulations can contain 1) algebraic equations for an unknown coefficient (see, for example, [2], 2) differential equations for an unknown coefficient (see, for example, [4]), 3) integral equations for an unknown coefficient [1]. The first case is the simplest and allows to restore the unknown function only at those points through which the reaction front passed during its experimental observation. In this case, a point-wise recovering of the unknown coefficient is possible. The second case is more complicated, since additional input information is required for the correct formulation of the inverse problem being solved. The third case is the most difficult.

Often the methods of asymptotic analysis are inapplicable for solving inverse problems of this type (see, for example, [3]). In this case the methods based on minimizing the target functional by the gradient method are used to sole the inverse problem under consideration.

Thus, the features of numerical reconstruction of some coefficients in solving the coefficient inverse problem for a nonlinear singularly perturbed equation of the reaction-diffusion-advection type will be discussed.

References:

[1] Lukyanenko D.V., Borzunov A.A., Shishlenin M.A. Solving coefficient inverse problems for nonlinear singularly perturbed equations of the reaction-diffusion-advection type with data on the position of a reaction front, Communications in Nonlinear Science and Numerical Simulation, Vol. 99, No. 105824 , 2021.

[2] Lukyanenko D.V., Shishlenin M.A., Volkov V.T., Asymptotic analysis of solving an inverse boundary value problem for a nonlinear singularly perturbed time-periodic reaction-diffusion-advection equation, Journal of Inverse and Ill-Posed Problems, Vol. 27, No. 5, 745-758, 2019.

[3] Lukyanenko D., Yeleskina T., Prigorniy I., Isaev T., Borzunov A., Shishlenin M., Inverse problem of recovering the initial condition for a nonlinear equation of the reaction-diffusion-advection type by data given on the position of a reaction front with a time delay, Mathematics, Vol. 9, No. 4, 342, 2021.

[4] Levashova N., Gorbachev A., Argun R., Lukyanenko D., The problem of the non-uniqueness of the solution to the inverse problem of recovering the symmetric states of a bistable medium with data on the position of an autowave front, Symmetry, Vol. 13, No. 5, 680, 2021.