

**积分方程数值分析研讨会**

Workshop on

Numerical Analysis of Integral Equations

**主办单位：**深圳北理莫斯科大学

计算数学与控制系

莫大-北理工-深北莫应用数学联合研究中心

2025年4月30日·深圳

**一、会议日程(Program Schedule)**

|  |  |
| --- | --- |
| **第一天(Day 1): April 30, 2025** | |
| **Chair: Ye Zhang** | |
| 9:30-10:00 | Ye Zhang  Introduction to Shenzhen MSU-BIT University |
| 10:00-11:00 | Hanxiao Wang  Solvability of coupled forward-backward Volterra integral equa  tions |
| 11:00-12:00 | Hecong Gao  Discontinuous Galerkin methods for index-1 and index-2 integral  algebraic equations |
| **Afternoon Session** | |
| **Chair: Ye Zhang** | |
| 14:30-15:30 | Chenyao Wang  Multi-task learning enhenced deep neural networks for delay dif  ferential equations |
| 15:30-16:30 | Haiyan Zhang  On a mixed Galerkin method for semi-explicit index-1 integro  differential algebraic equations |
| 16:30-17:30 | Free discussion |

**二、报告摘要(Abstracts)**

**Hanxiao Wang, Shenzhen University, China,**

**Title: Solvability of coupled forward-backward Volterra integral equations**

Abstract: Motivated by the optimality system associated with controlled (forward) Volterra integral equations (FVIEs, for short), the well-posedness of coupled forward-backward Voterra integral equations (FBVIEs, for short) is studied. The main feature of FBVIEs is that the unknown {(X(t, s), Y (t, s))} has two arguments. By taking t as a parameter and s as a (time) variable, one can regard FBVIE as a system of ordinary differential equations (ODEs, for short), with infinite-dimensional space values {(X(·, s), Y (·, s)); s ∈ [0, T]}. To establish the well-posedness of such an FBVIE, a new non-local monotonicity condition is introduced, by which a bridge in infinite-dimensional spaces is constructed. Then by generalizing the method of continuation developed by [Hu-Peng1995, Yong1997, Peng-Wu1999] for differential equations, we have established the well-posedness of FBVIEs. Joint work with Wenyang Li and Jiongmin Yong.

**Hecong Gao, Harbin Institute of Technology, Shenzhen, China,**

**Title: Discontinuous Galerkin methods for index-1 and index-2 integral algebraic equations**

Abstract: The integral-algebraic equation (IAE) of index 1 is a mixed system of first-kind and second-kind Volterra integral equations (VIEs). The discontinuous 11Galerkin (DG) method is proposed to solve the index-1 IAE, and the optimal global convergence order is obtained. The iterated DG method is introduced in order to improve the numerical accuracy, and the global superconvergence of the iterated DG solution is derived.

The discontinuous Galerkin method is further employed to solve index-2 integralalgebraic equations. First, the convergence theory of perturbed DG methods for first-kind VIEs is established, and then used to derive the optimal convergence properties of DG methods for index-2 IAEs. It is shown that an (m − 1)-th degree DG approximation exhibits global convergence of order m when m is odd, and of order m − 1 when m is even, for the first component x1 of the exact solution, corresponding to the second-kind VIE, whereas the convergence order is reduced by two for the second component x2 of the exact solution, corresponding to the first-kind VIE. Each component also exhibits local superconvergence of one order higher when m is even. When m is odd, superconvergence occurs only if x1 satisfies x ( 1 m) (0) = 0. Moreover, with this condition, we can extend the local superconvergence result for x2 to global superconvergence when m is odd. Note that in the DG method for an index-1 IAE, generally, the global superconvergence of the exact solution component corresponding to the second-kind VIE can only be obtained by iteration. However, we can get superconvergence for all components of the exact solution of the index-2

IAE directly.

**Chenyao Wang, Harbin Institute of Technology, Shenzhen, China,**

**Title: Multi-task learning enhenced deep neural networks for delay differential equations**

Abstract: We present a novel multi-task learning (MTL) enhenced DNN to solve both forward and inverse problems of delay differential equations (DDEs). The core idea behind this approach is to capture information at primary discontinuity points by employing a MTL enhenced DNN that can seamlessly integrate the regularity at primary discontinuity points into the loss function. Subsequently, the network is trained task by task, and transfer learning is applied to task-specific parameters of the network across tasks to expedite network convergence, thereby reducing the complexity of network training and offering reference solutions for subsequent tasks. The MTL enhenced DNN with the sequential training scheme significantly improves approximation accuracy and computation efficiency, which is demonstrated by solving several problems involving time-dependent DDEs and spatio-temporal DDEs,12contrasting its performance with that of the conventional DNN method.

**Haiyan Zhang, Harbin Institute of Technology, Shenzhen, China,**

**Title: On a mixed Galerkin method for semi-explicit index-1 integro**

**differential algebraic equations**

Abstract: The semi-explicit index-1 integro-differential algebraic equation (IDAE) is a coupled system of Volterra integro-differential equations (VIDEs) and second kind Volterra integral equations (VIEs). The existence, uniqueness and regularity of the exact solution are analyzed in detail. A numerical scheme mixed continuous Galerkin (CG) and discontinuous Galerkin (DG) method is proposed for the IDAE with the VIDE part approximated by CG schemes and the VIE part approximated by DG schemes. First, the global convergence order of the numerical solution is obtained, which is optimal for the VIE part, but not for the VIDE part. To improve the numerical accuracy, the iterated DG method is introduced for the VIE part. By virtue of the iterated DG method, the optimal global convergence is obtained for the VIDE part, and the global and local superconvergence results are gained for the new combination of numerical schemes with CG and iterated DG methods. Some numerical experiments are given to illustrate the theoretical results.